COMMENTS AND ADDENDA

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Excitonic Instability and Ultrasonic Attenuation in Strong Magnetic Fields

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Anomalies in ultrasonic attenuation in Bi under strong magnetic fields discovered by Mase *et al.* are interpreted as arising from the fluctuations of the order parameter of the excitonic phase above the critical temperature. The critical temperature is estimated to be of the order of 1° and the critical exponent of the excess contributions to the attenuation coefficient is $\frac{1}{2}$. These are in good agreement with the experiments.

Recently Mase and his collaborators[1] observed anomalies in ultrasonic attenuation in crystalline bismuth in strong magnetic fields. When the $N=1$ Landau level of electrons comes close to that of holes, the peak value of the absorption coefficient has a strong temperature dependence, which cannot be interpreted by the one-electron theory. We ascribe these anomalies to the fluctuations of the order parameter of the excitonic phase.

We adopt a simple two-band model for interacting electrons (a band) and holes (b band)[2] (effective masses are assumed to be equal and isotropic):

$$\mathcal{H}=\sum_\alpha \epsilon^a_\alpha\, a^\dagger_\alpha a_\alpha+\sum_\alpha \epsilon^b_\alpha\, b^\dagger_\alpha b_\alpha + V\int d\vec{r}\,\psi^\dagger_a\psi^\dagger_b\psi_b\psi_a\ , \quad (1)$$

where

$$\psi_a=\sum_\alpha\chi_\alpha a_\alpha,\quad \psi_b=\sum_\alpha\chi_\alpha b_\alpha\ , \quad (2)$$

$$\epsilon^a_\alpha=(N+\tfrac{1}{2})\omega_c+p^2/2m+E_a\ ,$$
$$\epsilon^b_\alpha=-\left[(N+\tfrac{1}{2})\omega_c+p^2/2m+E_b\right]\ . \quad (3)$$

In Eq. (2) we have,

$$\omega_c=1/ml^2\ \ (l^2=c/|e|H)\ ,$$
$$\chi_\alpha\equiv\chi_{NpX}=(1/L)\exp[i(X/l^2)y-ipz]\,\varphi_N(x-X)\ , \quad (4)$$

where ϕ_N is Nth eigenfunction of the linear harmonic oscillator and E_a and E_b are energies that arise from spin Zeeman terms. We take $\hbar=1$, and the momentum are measured relative to each band extremum.

If we increase the strength of magnetic fields with fixed orientations, one of the Landau levels with $N=1$ (of electrons) crosses over the Fermi energy, which corresponds to the peak of the absorption coefficient[3] in the lower-field side followed closely by another peak due to holes.

The Coulomb interaction in Eq. (1) is approximated to be of short range and the magnitude is of the order of

$$V=4\pi e^2/\epsilon_0\kappa^2\ , \quad (5)$$

where ϵ_0 is the static dielectric constant and κ is the inverse of the screening radius in the presence of strong magnetic fields. At low temperatures, where $T\ll\omega_c$, κ^2 is determined almost by the $N=1$ Landau levels as these lie near the Fermi level, and is estimated by the random phase approximation (RPA) to be ($k_B=1$)

$$\kappa^2=[\sqrt{2}\,me^2/(2\pi\epsilon_0 l)]\,(\omega_c/T)^{1/2}\ . \quad (6)$$

Here, for convenience sake, we restricted our considerations to the case of pure samples. Although

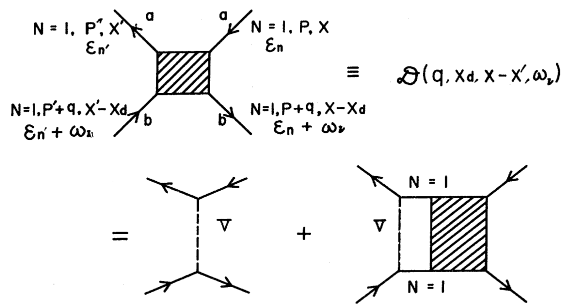

FIG. 1. Approximation for the fluctuation propagator.

(6) is divergent as $T\to 0$, in reality the impurity scattering removes this divergence. From a theoretical standpoint, self-consistent treatment is, of course, necessary, but we take κ^2 as a constant independent of temperature whose magnitude is given by (6) with $(\omega_c/T)^{1/2}$ replaced by a number of order of 10.

Excitonic instability[4] is represented diagrammatically by the divergence of the fluctuation propagators defined by Fig. 1. The explicit dependence of $\mathfrak{D}$ on $(X-X')$ comes from the fact that although the interaction itself is of short range, matrix elements are to be taken with respect to functions (4) with finite extension of the wave packet. Transforming $\mathfrak{D}$ in the Fourier integral

$$D(q, X_d, Q; \omega_\nu)\equiv(1/2\pi)\int_{-\infty}^{\infty} dX\, e^{iQX}\,\mathfrak{D}(q, X_d, X; \omega_\nu)\ , \tag{7}$$

we can solve Fig. 1 and get for $N=1$

$$D^{-1}=-\left[1/JV+\Pi(q,\omega_\nu)\right]\ , \tag{8}$$

where

$$J\equiv J(X_d, Q)=e^{-l^2q_\perp^2/2}\left[\tfrac{1}{2}(1-l^2q_\perp^2)\right]^2\ ,\qquad l^2q_\perp^2=(X_d/l)^2+(lQ)^2\ ; \tag{9}$$

and

$$\Pi(q,\omega_\nu)=\sum_\alpha \frac{f(\xi_a(p+q))-f(\xi_b(p))}{\xi_a(p+q)-\xi_b(p)-i\omega_\nu}\ ,$$
$$\xi_a(p)=\tfrac{3}{2}\omega_c+p^2/2m+E_a\ , \tag{10}$$
$$\xi_b(p)=-(\tfrac{3}{2}\omega_c+p^2/2m+E_b)\ .$$

The energies are measured with respect to the chemical potential. Critical temperature is determined by the equation

$$D(0,0,0;0)^{-1}=0\ . \tag{11}$$

To see the order of magnitude of T_c given by (11), we assume G, defined by

$$G=-\tfrac{1}{2}(3\omega_c+E_a+E_b)\ ,$$

is equal to zero and get

$$T_c=\frac{\omega_c}{2\pi}\left(\frac{V}{\omega_c l^3}\,\frac{2\sqrt{2}-1}{4\pi^2}\,\zeta(\tfrac{3}{2})\right)^2\ , \tag{12}$$

where $\zeta(x)$ is Riemann's ζ function. Taking the mass ratio as 0.01 and using (5) with a temperature independent κ^2, we have

$$T_c=5\ {}^\circ\mathrm{K} \tag{13}$$

for $\omega_c=100\ {}^\circ\mathrm{K}$. Equation (13) is really an overestimation due to the replacement of the screened Coulomb interaction by its maximum value. With a more elaborate theory we may get T_c of the order of magnitude of 1° in the system with strong magnetic fields. For a general value of G, we calculated (10) numerically, adopting V as a parameter which makes $T_c=1\ {}^\circ\mathrm{K}$ for $G=0$ and $\omega_c=100\ {}^\circ\mathrm{K}$. Figure 2 shows T_c as a function of $|g|$ $(g=G/\omega_0)$. $G>0$ and $G<0$ correspond to the lower- and higher-field sides, respectively. This figure clearly shows that T_c is always higher for $G>0$ than for $G<0$, and so at fixed temperature, effects of fluctuations are more evident in the lower-field side. Note that T_c for $G>0$ has a termination point, which is due to the fact that the polarization function $-\Pi(q,\omega_\nu)$ has a maximum value as a function of T if $G>0$. Near the critical temperature T_c we can expand D^{-1} and get

$$D(q, X_d, Q; \omega_\nu)^{-1}=-(E/16\sqrt{2}\pi^2|g|^{1/2}\omega_c l^3)\left[\eta+D_\perp(lq_\perp)^2+D_\parallel(2|g|)^{-1}(lq)^2+(|\omega_\nu|/T)\,\theta(G)F\right]\ , \tag{14}$$

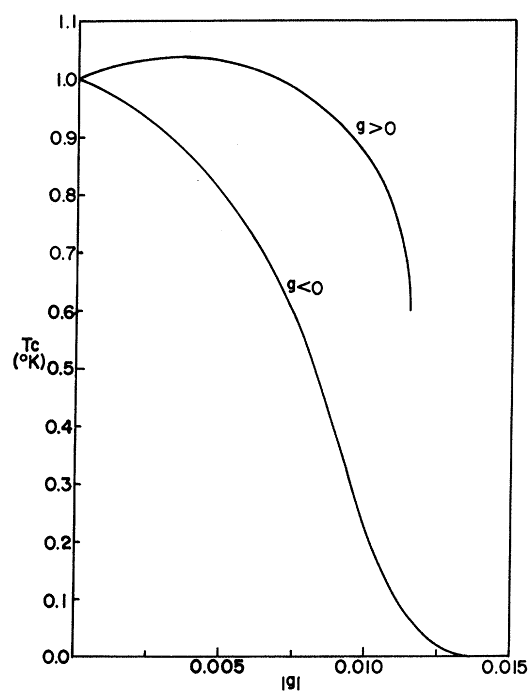

FIG. 2. Critical temperature for the lower-field side ($g>0$) and the higher-field side ($g<0$).

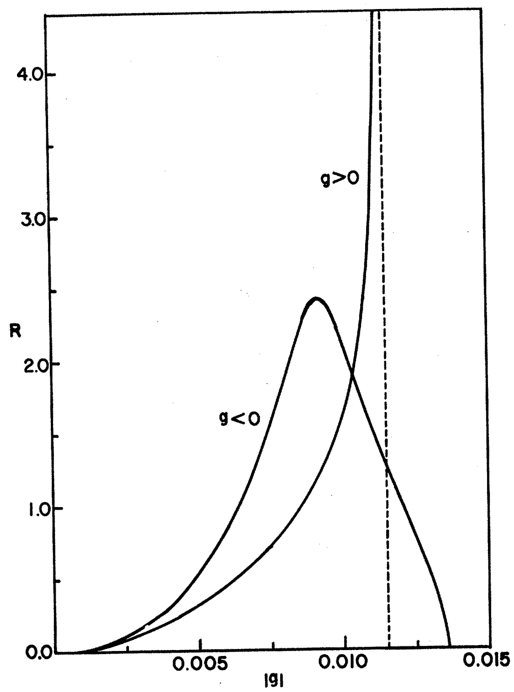

FIG. 3. R defined by Eq. (15) for the lower-field side ($g>0$) and the higher-field side ($g<0$).

where $\eta=(T-T_c)/T_c$, $\gamma_c=G/T_c$, and E, $D_\perp$, $D_\parallel$, and F are functions of γ_c of the order of 1.

As regards ultrasonic attenuation,[3] due to the three-dimensional character of the fluctuation propagator D [Eq. (14)] terms which correspond to Azlamazov-Larkin (AL) diagram[5] in fluctuating superconductors yield a singular temperature dependence for the phonon self-energy function. After straightforward calculations, we get an excess contribution $\Delta\alpha$ to the attenuation coefficient,

$$\Delta\alpha/\alpha_n=R(\gamma_c)\eta^{-1/2}, \qquad (15)$$

where α_n is the peak value of the attenuation coefficient in one-electron theory. As R has rather complicated expressions, we show $R(\gamma_c)$ numerically in Fig. 3 as a function of g. The vanishing contribution of R when $G=0$ is due to the complete cancellation of contributions from electrons and holes whose effective masses are assumed to be equal. If $G\neq 0$, the heat absorbed by one kind of particle, electrons or holes, of the pair is transfered to the other by the correlations. Asymmetry of the effects of the anomaly between the low- and high-field sides of peaks is clear from Fig. 2.

Such many-body effects are expected to be important for some other quantities, such as the Hall coefficient and electrical conductivity.

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[1]T. Sakai, Y. Matsumoto, and S. Mase, J. Phys. Soc. Japan 27, 862 (1969); Y. Matsumoto, T. Sakai, and S. Mase, *ibid.* 28, 1211 (1970); S. Mase, Y. Matsumoto, T. Sakai, and Y. Suido, in *Proceedings of the Twelfth International Conference on Low Temperature Physics, Kyoto*, 1970 (Academic Press of Japan, Tokyo, to be published).

[2]Recently A. A. Abrikosov [J. Low-Temp. Phys. 2, 37 (1970); 2, 175 (1970)] discussed in detail excitonic instability in the presence of strong magnetic fields at zero temperature. He examined the vertex functions for the case where only the first Landau level is relevant, but we must, here, take into account the next Landau level. To do this we employed, as a gross approximation the random phase approximation.

[3]V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksperim. i Teor. Fiz. 40, 786 (1961) [Sov. Phys. JETP 13, 552 (1961)].

[4]For example, see D. Jerome, T. M. Rice, and W. Kohn, Phys. Rev. 158, 462 (1967) for cases in the absence of the magnetic field; and E. W. Fenton, Phys. Rev. 170, 816 (1968) for the case in the magnetic field.

[5]L. G. Azlamazov and A. I. Larkin, Fiz. Tverd. Tela 10, 1104 (1968) [Sov. Phys. Solid State 10, 875 (1968)].